Interaction of Astrodynamics and a Mars Imaging Mission

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Arrival geometry at the planet and orbit size significantly influence sensor field-of-view requirements, mapping altitude, and subvehicle illumination. Additional consideration of mapping time, planet coverage, allowable velocity, and data transmission capability result in a complex interaction of these parameters. The 1973 opportunity is taken for illustration, and the use of Type I trajectories is assumed. The present paper graphically displays these interactions with emphasis on the imaging experiment and telecommunication subsystem. Polar and inclined orbits are examined, and the effects of apsidal rotation and orbit inclination on illumination are discussed. Design charts that allow a rapid preliminary determination of sensor field-of-view and velocity requirements are presented. Auxiliary design charts determine the related bit rate and storage requirements, assuming a contiguous mapping mode. The data transmission capability of the telecommunication subsystem is expressed in terms of antenna size, power gain product, and subsystem weight.

I. Introduction

THE prime objective of an orbiter imaging mission is to obtain high resolution mapping photographs of as much of the planet's surface as possible, and as quickly as possible. There are also many important special objectives, such as the observation of unique and localized surface features, the use of stereo techniques, and the generation of multispectral images of surface features. However, these objectives will probably be secondary to the prime mapping mission, at least for the first orbiting missions. Orbiter imaging objectives are major contributors to a high mission science value, even for missions having a primary landing objective. This paper discusses the interaction of astrodynamical and subsystem parameters for the prime orbiter imaging mission. Emphasis is placed on interconnecting design parameters between astrodynamics and the imaging and telecommunications subsystems. By performing interrelated design analysis, certain preliminary design and flight criteria can be educed as also indicated in previous studies.[1,2,3] Some of these criteria include 1) selection of candidate orbits, 2) specification of imaging optics, 3) specification of telecommunication subsystem characteristics, and 4) specification of velocity requirements.

For purposes of illustration, the 1973 Mars mission opportunity is taken, and the use of Type I trajectories is assumed. The astrodynamical considerations of an orbiter imaging mission will be discussed in terms of the imaging data quality, quantity, and the ability to transmit the data to earth. The quality of the imaging data is a function of the surface resolution and illumination. High resolution can be obtained with the least imaging optics weight at the lower mapping altitudes. Illumination at oblique sun angles is required to detect many surface details characterized by changes in slope or elevation. For the purposes of this paper, the criteria of low altitude mapping at sun angles between 20° and 70° will be assumed as a measure of high data quality.

The quantity of the imaging data, as measured in information bits, is a function of the resolution size and the total surface area viewed. Since the resolution size is determined by the imaging sensor, this section of the paper will discuss only the ability of the orbit to permit maximum surface viewing, in terms of percent of total planet surface, in the shortest time.

The amount of imaging data that can be usefully collected is limited by the ability of the telecommunication subsystem to transmit it to earth. This section of the paper will describe the restraints placed on the imaging resolution and the rate of surface mapping by the data transmission rate and data storage rate. The data transmission rate will be expressed in terms of antenna size, transmitter power level requirements, and subsystem weight.

II. Data Quality

Launch and arrival dates are the most important parameters affecting subvehicle illumination, since, for a given orbit inclination, they initially establish the orientation of the orbit plane relative to the terminator plane. After arrival, the illumination history depends on planet oblateness effects on the spacecraft orbit and the Mars orbit around the sun. For early orbiting missions, it is expected that apoapsis altitudes will be greater than 10,000 km in order to realize large in-orbit useful weights. Figure 1 pictorially shows a polar orbit for an early February arrival case. Initially, the subperiapsis point will be in darkness, thereby making it necessary to conduct imaging tasks at altitudes significantly higher than periapsis altitude. Figure 2 shows the subvehicle illumination map for a polar orbit and an arrival date of February 22, 1974. Note that the periapsis latitude is approximately 50°N at arrival. The daylight latitude regions bounded by the morning and evening terminators are shown for orbital passes occurring a number of days after encounter. Subvehicle illumination is given by the illumination angle (ζ) where an angle of 90° denotes a terminator crossing. For the sample case shown in Fig. 2, the best imaging data is obtained within 25 days after en-

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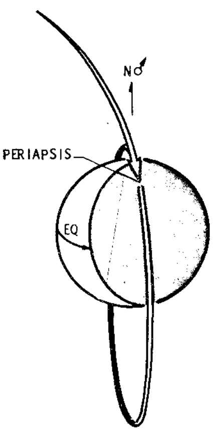

Fig. 1. Polar orbit arrival.

counter. This quality data, however, is limited to ±50° latitude zone, and imaging tasks are conducted from altitudes of 4000 km to 16,000 km. After 90 days, the subvehicle illumination improves again, and mapping altitudes are on the order of 1000 km to 2000 km. With fixed optics, mapping from two different altitudes results in taking data at two different resolutions. If both wide and narrow angle optics are on board, it may be advantageous to use the narrow angle optics for the wide angle mapping function at high altitudes. Mapping altitudes can be decreased if a negative apsidal rotation is incorporated in the orbit insertion maneuver. Apsidal rotation is defined as the angular displacement of the areocentric orbit periapsis from the periapsis of the approach hyperbola, and is measured positive in the forward flight direction. A 40° rotation is equivalent to shifting the altitude contours in Fig. 2 an equivalent amount on the spacecraft latitude scale. Mapping altitudes of 14,000 km can be reduced to 6000 km in the early portion of the mission and will increase from 1500 km to 4000 km at 90 days after encounter.

Arrival at a later date, for example, April 1, 1974, will initially place the subperiapsis point in daylight but within 10° of the evening terminator. Initial mapping altitudes for the first 30 days will be less than 2000 km, and the illumination and latitude coverage improves with time. In examining Earth-to-Mars trajectory characteristics, later arrivals in April 1974 will require increasing launch energy

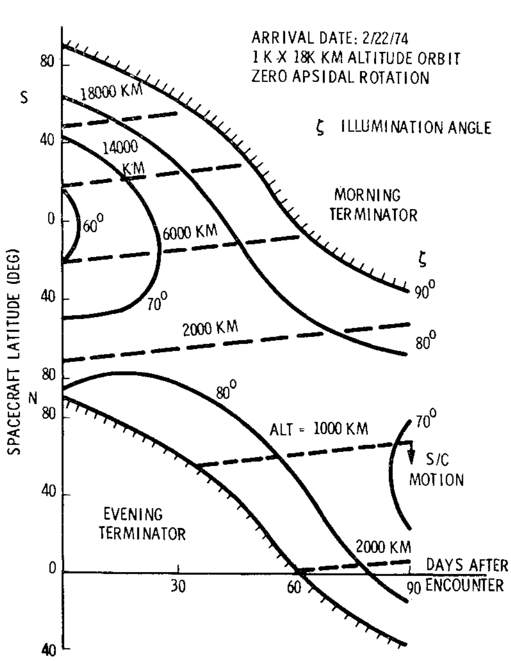

Fig. 2 Subvehicle illumination map (inclination = 90°).

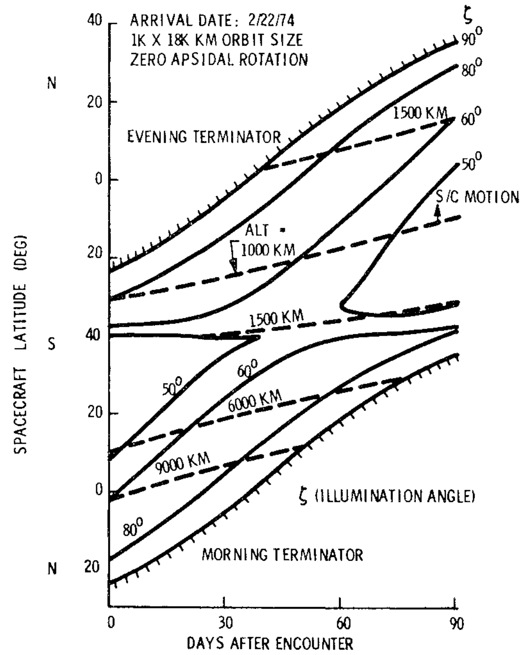

Fig. 3 Subvehicle illumination map (inclination = 40°).

to generate a launch window.[4] Earlier arrivals in January 1974 result in increasing orbit insertion velocity requirements.

Improvement in mapping conditions can be achieved with a lower inclined orbit of 40°, as shown in Fig. 3. However, the latitude region covered will be limited for the most part to the southern hemisphere early in the mission, since the hyperbolic approach trajectory is from the south. For this example, mapping altitudes are between 1000 km and 2000 km and remain at these values for over 90 days of mission time. In order to map the northern hemisphere at low altitudes, the mission time would have to be extended beyond 90 days. The order of hemispheric coverage can be reversed if the approach trajectory is from the north. With inclined orbits, planet coverage is restricted to a north and south maximum latitude, equivalent to the value of orbit inclination plus one-half the imaging swath width.

One method that has been suggested to improve mapping conditions for early arrival dates, while still achieving a high inclination, uses an orbit plane change maneuver, as illustrated in Fig. 4. The spacecraft approaches the planet at a lower inclination (30–40°) and performs the orbit insertion maneuver that results in a predetermined apsidal rotation. At a subsequent apoapsis passage, a plane change maneuver is executed which increases the inclination.

A similar plane change maneuver can also be incorporated in the orbit insertion maneuver, however this method will not be discussed here.

The illumination map associated with the former maneuver sequence is presented in Fig. 5 for a final orbit inclination of 90°. It is noted that good illumination extends over large portions of both hemispheres. The initial subperiapsis point

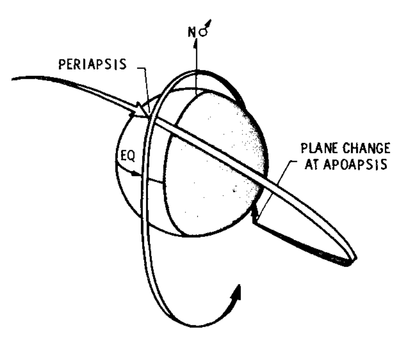

Fig. 4 Inclined arrival with plane change.

is now placed in daylight and at a latitude of 20°N. Mapping altitudes can be restricted to less than 3000 km, covering latitude regions between +90° and −50°. The attendant velocity cost for the plane change maneuver is approximately 0.6 km/sec.

III. Data Quantity

The quantity of imaging data that can be potentially obtained is dependent on the surface area overflown and the surface swath width. Swath width is the width of surface coverage swept by the imaging sensor field-of-view while in orbit, and is expressed in terms of surface central angle. The swath width requirement is determined by the parameters a) mapping time, b) longitudinal displacement of neighboring ground track, c) effective inclination, and d) desired overlap of neighboring swath.

For a desired mapping time, the semi-major axis that results in equally spaced equatorial crossings can be determined through an iterative solution of the following equation:

$$2\pi\left(\frac{a^3}{\mu}\right)^{1/2}\left[1 - J\left(\frac{R}{a(1-e^2)}\right)^2\left(\frac{7\cos^2 i - 1}{4}\right)\right] = \frac{360}{\omega}\left[\frac{1}{j-(k/N)} - J\left(\frac{R}{a(1-e^2)}\right)^2\cos i\right] \quad (1)$$

where μ = gravitational constant, R = Mars radius, ω = Mars rotation rate, J = 2nd zonal harmonic, i = orbit inclination, e = orbit eccentricity, and a = semi-major axis.

Specifically, a is the semi-major axis that determines the orbit period (P) required to obtain a repetitive uniform coverage pattern in N Martian days. Orbit size is controlled by the integer j, and the skip (or crossing control) parameter k governs the relative location of the nearly repeating ground tracks. The longitudinal displacement of neighboring ground track (σ) is then found by:

$$\sigma = (P\omega \pm \Delta\Omega)/N \quad (2)$$

where $\Delta\Omega$ is the longitudinal regression of the orbit plane with respect to Mars per orbit revolution.

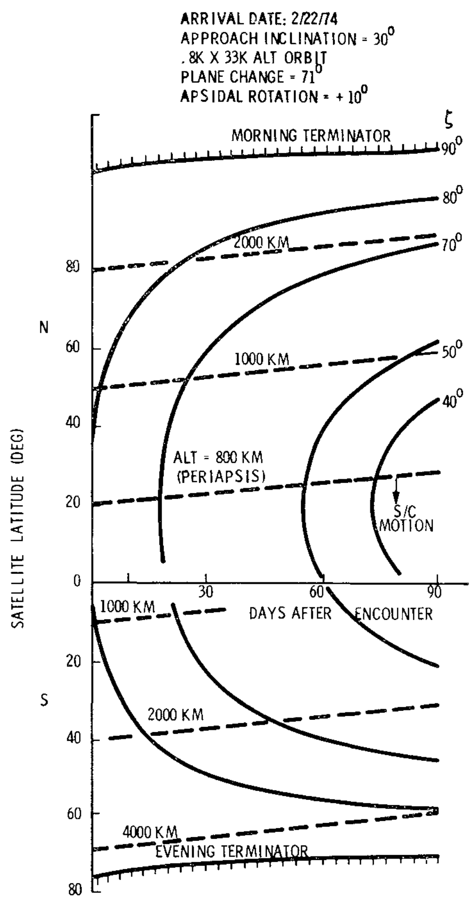

Fig. 5 Subvehicle illumination map with plane change (inclination = 90°).

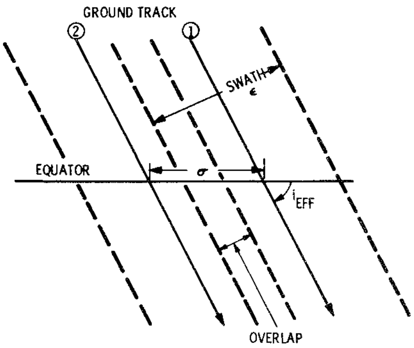

Fig. 6 Imaging geometry.

Figure 6 depicts the longitudinal displacement of two neighboring ground tracks at the equatorial crossing. Because of the rotation of Mars, and possible high orbital altitudes, the resulting ground track may have an effective inclination much different from the true orbit inclination. The effective inclination at the equator is given by Eq. (3)

$$i_{\text{eff}} = \tan^{-1}[V_N \sin i/(V_N \cos i - \omega)] \quad (3)$$

where V_N = the angular velocity.

Assuming that the imaging swath is as shown in Fig. 6, the swath width required to achieve a desired overlap can be determined by:

$$\epsilon = 2\sin^{-1}(\sin i_{\text{eff}}.\sin\sigma/2)/(1-\lambda) \quad (4)$$

where ϵ = the required swath width, and λ = fractional overlap.

The sensor field-of-view necessary to achieve the required swath width from a given altitude (h) can be computed from:

$$\delta = 2\tan^{-1}\left\{\frac{R\sin[(\sigma)/2(1-\lambda)]}{(R+h) - R\cos[(\sigma)/2(1-\lambda)]}\right\} \quad (5)$$

Since early Mars missions will likely use highly elliptical orbits, determination of field-of-view requirements will become more difficult. Sizing of optics for equatorial crossings can result in large mapping "gaps" at significantly better resolution in certain latitude regions. These data gaps are caused by the rapid decrease in altitude for those cases where the equatorial daylight altitude is considerably greater than the periapsis altitude. Compensation can be made to eliminate these imaging gaps, but with an accompanied increase of imaging overlap in other latitude regions. Therefore, the definition of overlap is less meaningful, since its constancy is less controllable for these cases. Design of nonvariable optics results primarily in a variation of resolution with altitude.

Studies conducted by the authors have indicated that selecting a sensor field-of-view nearly equivalent to the value required to eliminate imaging gaps results approximately in a maximum value return. Figure 7 demonstrates how the astrodynamical parameters (arrival date, orbit size, apsidal rotation, inclination) and mission specifications (mapping time, imaging overlap) affect sensor field-of-view selection.

Data presented in Fig. 7 is for a polar orbit and uses the interconnecting variable of equatorial daylight altitude to relate mapping time, orbit size, arrival date and apsidal rotation to sensor field-of-view requirement. Periapsis altitude is taken to be 1000 km. In using the design chart, a

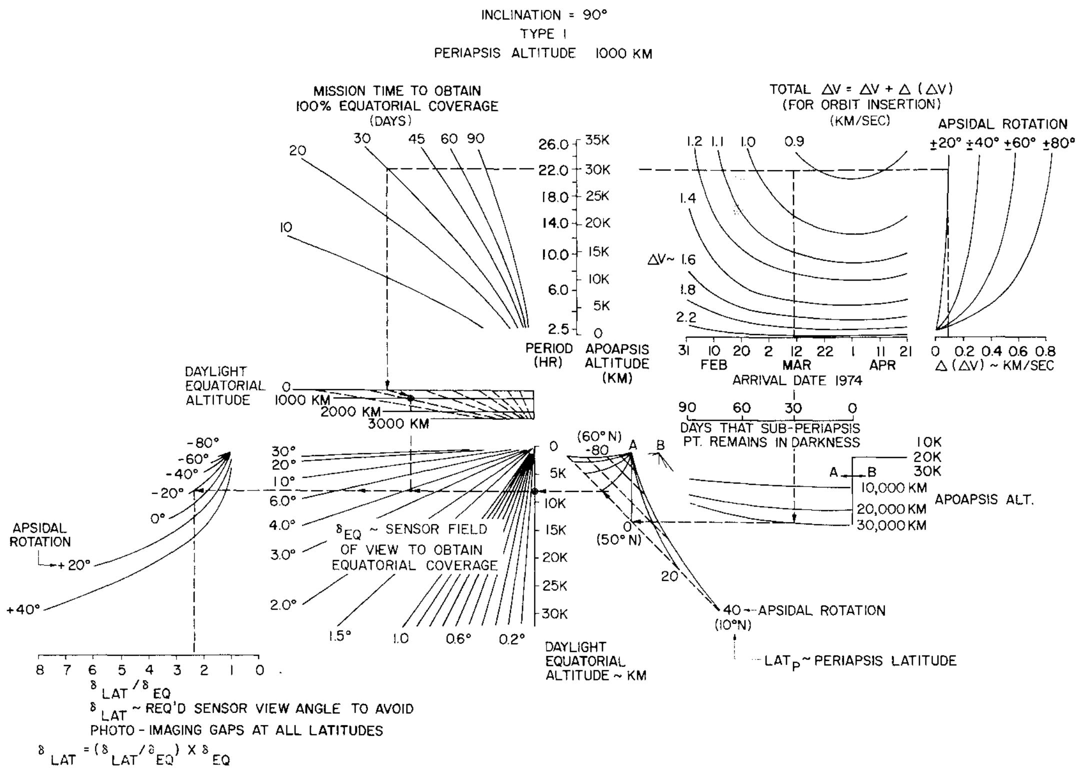

Fig. 7 1973 mapping parameters (inclination = 90°).

desired mapping time, apoapsis altitude (or orbital period), and an arrival date can be assumed. The sample included in Fig. 7 assumes a mapping time of 30 days, an apoapsis altitude of 30,000 km, and an arrival date of March 11, 1974. The first objective is to determine the equatorial altitude This is accomplished by proceeding vertically downward from the arrival date scale to the respective apoapsis altitude value, then horizontally to the base line (zero apsidal rotation) of the apsidal rotation correction curves. Note that velocity requirements can also be determined. Explanation of these additional velocity graphs are discussed in a subsequent section. Dashed lines in the intermediate correction curves are guidelines. If there is no apsidal rotation, then the equatorial daylight altitude is directly read (e.g., 14,000 km for zero apsidal rotation). In the example, the guidelines are used to proceed from the baseline to the desired apsidal rotation value of $-20°$. The equatorial daylight altitude is then read as 8000 km. As indicated in the figure, the periapsis latitude will be approximately 70°N. Having found the value of equatorial daylight altitude, proceed vertically downward from the intercept of the specified apoapsis and mapping time to the base line (0 km) of equatorial daylight altitude curves. These curves are really correction factors to swath width for the variation in effective inclination. After intercepting the daylight equatorial altitude baseline, the guidelines are again followed to the previously determined equatorial daylight altitude of 8000 km. By continuing vertically downward to an intersection with 8000 km value from the equatorial daylight altitude scale, the required sensor field-of-view is found to be 4°. This field-of-view requirement represents that which is necessary to obtain 100% equatorial coverage within the desired mapping time and without overlap. If avoidance of imaging gaps in other latitude regions is desired, then the 4° field-of-view must be increased by a factor of 2.4 to a field-of-view of 9.6°. The necessary correction factor is also given as δ_{LAT}/δ_{EQ} in Fig. 7.

Similar charts of interacting parameters can be prepared for other orbit inclinations or maneuver strategies. It is important to note here that the percent of planetary coverage can be initially increased significantly for inclined orbits by oversizing the sensor field-of-view.[5] This action is equivalent to increasing orbit inclination but leads to greater transmission rate requirements.

Trends noted are that several field-of-view requirements may be necessary during the mission, and that increasing mapping time or decreasing orbit period generally reduces the sensor field-of-view requirement. Furthermore, apsidal rotation can significantly influence daylight equatorial altitude and consequently, field-of-view.

If the sensor field-of-view required to eliminate all possible imaging gaps cannot fully be mechanized, the planet coverage can be less than that available for a given inclination. Empirical investigations have shown that the percent of available planet coverage can be approximately estimated for all arrival dates, orbit sizes, and apsidal rotation by normalizing the actual sensor field-of-view to the field-of-view requirement that eliminates imaging gaps. Available coverage is defined as the illuminated latitude band that can be mapped from a given orbit inclination. This is shown in Fig. 8 for the 90° and 45° inclined orbits. The percent of available planetary coverage varies with the sine of orbit inclination. This figure shows that the sensor field-of-view requirement can be reduced by 20%, if necessary, with only a 10% loss in the available coverage. Although less than the possible coverage may be obtained, it can be expected that large quantities of overlap imaging will still be accumulated.

Velocity Requirements

Figure 7, also includes data that allow preliminary estimates of velocity requirements. The velocity data has been prepared by assuming a constant planet approach velocity for a given arrival date. The velocity requirements for the polar orbit and any other inclined orbits are identical. By selecting an apoapsis altitude and arrival date, the orbit insertion velocity requirement can be estimated. If an apsidal rotation is included at orbit insertion, the additional velocity allowance, $\Delta(\Delta V)$, may be found and added to the orbit insertion requirement to determine the total velocity requirement. For the cases where plane change maneuvers are involved, velocity requirements can be expected to be greater. Plane change maneuvers, either to enhance mapping coverage or to improve capsule impact conditions (landing missions), will generally be of smaller magnitude for the April 1974 arrival dates where initial coverage and landings occur near the evening terminator. Velocity adjustments for midcourse corrections and orbit trim must also be considered as they occur in the mission profile.

Engineering Considerations

In addition to considering velocity limitations that might be imposed, other engineering considerations must be included in selecting candidate orbits and sensor field-of-view requirements. These include mission objectives and various occultation criteria. For example, landing missions may specify capsule landing locations and communication geometry, which in turn may dictate arrival date, trajectory type, inclination, and apsidal rotation. The restricted choice in candidate orbits may be in direct opposition to orbital imaging goals. Investigations of Canopus, solar, and earth occultations have also indicated restrictiveness in orbit selection.[6] Usually, solar occultations are not a problem unless the orbit inclination is less than 20°. Canopus occultations pose a very severe restriction and are not easily avoided, except for those cases having low inclination (<30°) and small apsidal rotation (<30°). Earth occultations at orbit insertion are usually encountered for retrograde orbits. In order to have a high-value return, it may become necessary to compromise some of the engineering constraints and science requirements. The best compromise can be evaluated by means of value analyses similar to those demonstrated by Kennet et al.[7]

IV. Data Return

The ability to accommodate and transmit the quantity of imaging data that can be obtained depends on the capability of the imaging and telecommunications subsystems. Imaging equipment consists of a lens system and imaging sensors (e.g., vidicon or film.). The telecommunications subsystem records, processes, stores, and transmits the data to earth stations. The recorder input bandwidth (BW) of the imaging sensor analog signal is a function of the spacecraft altitude (H), relative surface velocity (V), desired resolution (r), and sensor field-of-view (δ). For contiguous coverage, the required bandwidth can be computed with the following equation, which assumes an image format compatible with the spatial resolution of the imaging system and a nonrotating planet.

$$BW = 2HV \tan(\delta/2)/r^2K^2v \tag{6}$$

The corrections for scan line time and effective visual resolution are given by v and K (Kell factor), respectively. Figure 9 presents an imaging design chart, a section of which is implemented by Eq. (6) using $K = 0.7$ and $v = 0.9$. If the sensor field-of-view is restricted because of recording rate

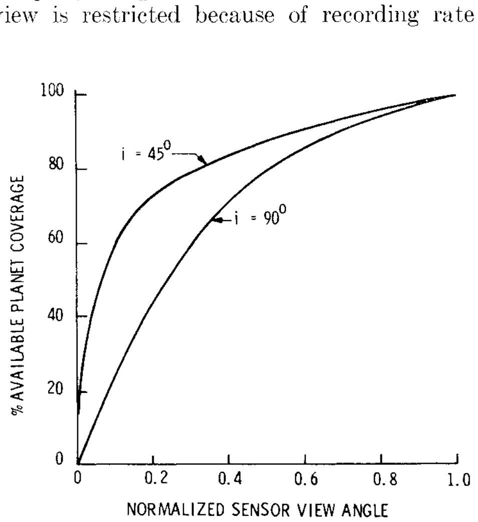

Fig. 8 Planetary coverage.

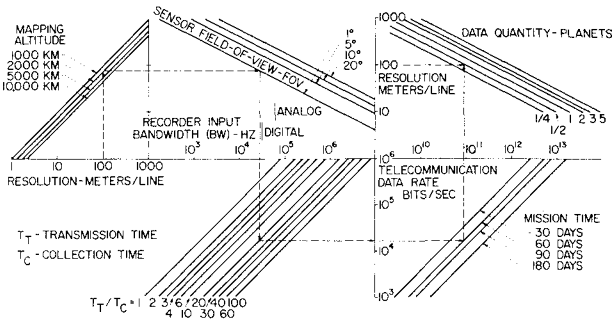

Fig. 9 Imaging design chart.

limitations, it may be necessary to have additional sensors to achieve an equivalent field-of-view.[5]

The required ratio of transmission time to collection time (T_T/T_C) can be estimated from illumination maps similar to those presented in Figs. 2, 3, and 5. The minimum value of this ratio, however, is dictated by the data transmission capability of the telecommunications subsystem. The larger this ratio (T_T/T_C), the more selective (or restrictive) is the imaging coverage. The data transmission rate requirement, shown in Fig. 9, assumes that two samples per cycle are taken, and each sample is quantized into 6 bits, corresponding to 64 gray levels. Storage capacity in bits (B) for the mission can be estimated by:

$$B = (12)(BW)(T_C/T_T)(P) \tag{7}$$

Figure 9 includes an illustration of a sample imaging system having a sensor field-of-view of 1°. The mapping altitude is 2000 km, and the desired ground resolution is 100 m. This results in a recorder bandwidth requirement of 3×10^4 Hz. Assuming a transmission to collection time ratio of 20, the transmission data rate required is 11×10^4 bits/sec. In a 60-day mapping mission, a total of 10^{11} bits would have been transmitted; this is an equivalent area coverage (not necessarily, percent of planet) of 0.50 planets. The fraction (F) of planet mapped in terms of data quantity is given by the following expression:

$$F = B(r^2)(K^2)/(8.772 \times 10^{14}) \tag{8}$$

The required storage capacity for these conditions and a 12-hour orbit is 77.7×10^7 bits. If the telecommunication system can only provide, for example, 9×10^3 bits/sec, then T_T/T_C must be increased to 40.

The telecommunications subsystem weight is mainly affected by the selection of data rate, but also influences other subsystems (e.g., additional attitude control gas to balance large antenna torques). Studies pertaining to the choice of data rate have been performed and are summarized in Fig. 10, a telecommunications subsystem design chart. The subsystem weight can be related to high gain antenna size, Traveling Wave Tube Amplifier (TWTA) size, power gain product, and arrival date. Arrival date (equivalently, communication distance) is the interconnecting astrodynamical parameter. The data presented in Fig. 10 assumes a bit error probability of 5×10^{-3}, error control coding, and a 210-ft receiving antenna with a worst case gain of 60 db and system temperature of 55°F.

Other subsystem influences on data rate which have been considered are: antenna size, efficiency, and weight; power amplifier (TWTA) weight and thermal control weight; attitude control gas to balance solar wind torque and antenna pointing inaccuracies resulting from alignments and mountings; control deadbands; and quantizations and curve fit errors.

The example shown in Fig. 10 assumes a communication distance of 200×10^6 km (late February arrival) and a telecommunication subsystem having a 20-watt TWTA and a

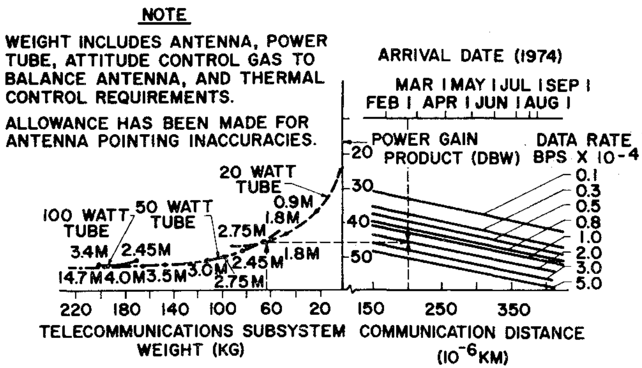

Fig. 10 Telecommunications subsystem design chart.

2.75-meter antenna. The data rate capability for this subsystem configuration is 2.0×10^4 bps, and the subsystem weight is 64 kg. Note that the same data rate capability can be obtained with a 50-watt TWTA and a 1.8-meter high gain antenna. Figures 10 and 9 can be used to establish preliminarily the quantity of data to be obtained, the intermediate storage capacity required, and telecommunication subsystem design requirements. Appropriate adjustments can be made for the different number and sizes of earth receiving antennas.

V. Concluding Remarks

It has been shown how system engineering methods can contribute to preliminary system and mission design. The case presented here treats an orbital imaging mission and has demonstrated how astrodynamical parameters influence the quality and quantity of imaging data. Most noteworthy is that methods of this type allow the mission analyst and systems engineer to rapidly select candidate orbits and subsystem design parameters. Knowledge of parameter interactions can easily suggest corrective actions and provide capability to anticipate experiment return value and subsystem performance.

More specific results concerning the 1973 mission with Type I trajectories are:

1) polar orbits provide only poor to fair illumination characteristics;

2) lower inclined orbits result in satisfactory lighting conditions, but at a loss in percent of planet coverage;

3) imaging missions using elliptic orbits can require several sensor field-of-view requirements during the mission;

4) use of plane-change maneuvers can enhance imaging quality and provide a high percent planet coverage, but with a significant velocity cost.

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